An Efficient Fixed-Point Realization of Inversion Based Repetitive Control*

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***Abstract*— The paper presents a fixed-point realization of non-minimum phase system inversion for inversion based control algorithms such as feedforward, repetitive controller, and iterative learning control. Based on the author's previous work on efficient dynamic inversion using stable pole-zero cancelation and non-minimum phase zero inversion using Kurosu filter, this paper employs the use of the Delta Operator filter realization to mitigate quantization effects in fixed-point computations. The Kurosu filter based inversion, where the difference of two IIR filters are used to approximate a high order FIR filter and switches are used to perform time reversals for phase compensation, is computationally efficient and well suited for low level fixed-point realization by digital signal processors or field programmable gate arrays. However, substantial quantization noise is evident in high sampling rate implementation. The Delta Operator realization increases computational complexity only slightly while providing substantial reduction of quantization noise. The method is employed in the feedforward and repetitive control of a piezoelectric actuator and the experimental results are presented to demonstrate its effectiveness.**

I. INTRODUCTION

Controller realization using field programmable gate arrays (FPGAs) has been of interest lately. A FPGA's low-level and low latency input-output interface and highly modularized parallel computation facilitates applications requiring a high sampling rate and/or a high channel count controller realization. However, the finite-word-length (FWL) and quantization issues in fixed point computation and limited number of computational resources make realization of more sophisticated high performance controllers challenging. High sampling rate repetitive control (RC) on an FPGA has proved helpful in cases of scanning probe microscopy [1], where simple discrete-delays are used to approximate plant phase delay for stable feedback performance. The delay compensation, although simple for a FPGA realization, may not be sufficient to compensate for phase delay of plants with complex dynamics. Techniques based on feedforward inversion, either zero phase error compensation (ZPETC) or approximate non-minimum phase system inversion [2]–[7], have shown effective for such situations. However, the FPGA based high-sampling rate implementation of these feedforward inversion based repetitive controllers are challenging due to computational complexity, proneness to saturate/overflow and quantization effect in the fixed point computation.

We have previously developed an efficient feedforward inversion based repetitive controller, which has been realized on an FPGA at $100kHZ$ sampling rate [8], [9]. The technique employed signal processing techniques of real-time infinite impulse response (IIR) linear phase filtering. The idea of linear phase IIR filtering is attractive at high sampling rates because the order, or computational complexity, remains relatively the same regardless of the sampling frequency. On the other hand, finite impulse response (FIR) controller or filters tend to increase in order as sampling rate increases for the same continuous-time impulse response. Powell and Chau first introduced a filter structure with memory management elements to provide a technique to perform online time-reversals to guarantee stability for linear phase IIR filters [10]. Kurosu slightly modified the structure to remove all non-linearities from the Powell-Chau filter to produce an *exactly* linear phase IIR filter [11]. Kurosu used the idea of subtracting two IIR filters to approximate a long FIR filter (controller). We have made use of the Kurosu's filter structure to produce a novel approximate non-minimum phase zero (NMPZ) inversion while keeping computational costs low. Furthermore, this non-minimum phase zero inversion was applied in inversion based repetitive control and implemented by an FPGA to control a piezoelectric actuator and a magnetically levitated shaft [8], [9].

A typical issue that plagues fixed-point realization of controllers is the issue of FWL effects. Given some coefficient truncation, phase and magnitude characteristics can be affected. The obvious fix of increasing the word length is often prohibitive in terms of both limited resources and/or timing restrictions. Given these constraints, the typical solution is to use different filter realizations, such as Direct Form II transposed (DFIIt), to ameliorate quantization noise and FWL effects [12]. Quantization noise becomes a lower bound to which an error control signal can achieve. With high sampling rate, high performance, FWL controllers, the levels of quantization noise can be substantial [8]. The Delta Operator is a filter realization specifically used for high sampling rates and to mitigate these FWL effects. Middleton and Goodwin first introduced the Delta Operator and studied FWL with and without the Delta Operator [13]. The advances with the Delta Operator have brought about methods in which the Delta Operator uses slightly more computational resources while providing substantial robustness against FWL effects [14], [15]. This paper investigates the performance benefit of using the Delta Operator versus the DFIIt realization of

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Kurosu's filter for the repetitive control of a piezo-electric actuator.

The rest of the paper is organized as follows. Section II reviews the approximate feedforward inversion of non-minimum phase zeros (NMPZ)s and Kurosu filter realization. Section III investigates the Delta Operator filter form. Section IV discusses the modeling of our system and how the Delta Operator affects the feedforward inversion and repetitive controller. Section IV-C contains experimental results on a piezoelectric actuated cutting tool, while highlighting the performance benefits of the Delta Operator filter form over the DFIIt filter form.

II. Kurosu-Based Inversion Filter

A. Background

Definition 1: Let FIR filters be denoted with a lower-case letter (e.g. $d(z)$) and IIR filters with a upper-case letter (e.g. $D(z)$).

Definition 2: Let $d^\star(z)$ and $D^\star(z)$ be defined as the time-reversal filter

$$d^\star(z) = d(z)|_{z=z^{-1}} \tag{1}$$

$$D^\star(z) = D(z)|_{z=z^{-1}} . \tag{2}$$

B. Kurosu Filter

Fig. 2 illustrates Kurosu's modified version of the Powell-Chau filter [10], [11]. We also require that $H(z)$ is causal and stable. Kurosu utilizes the fact that that any FIR filter can be represented as the subtraction of two IIR filters. Linear phase is achieved through the time reversals of the L-length localized time reversals through the use of Last-In-First-Out (LIFO) structures. Fig. 1 shows that L is typically chosen such that the truncated portion of the impulse response sits near quantization level. In this case, $H(z)$ and $H_L(z)$ are both IIR filters of same order, where $H_L(z)$ or $[\cdot]_L$ is the truncated or unwanted portion of the impulse response. $H(z)$ can be described as

$$H(z) = \frac{b_0 + b_1 z^{-1} + \ldots + b_{r-1} z^{-(r-1)} + b_r z^{-r}}{1 + a_1 z^{-1} + \ldots + a_{q-1} z^{-(q-1)} + a_q z^{-q}} \tag{3}$$

in its filter form. Also $q \geq r$, $b_i, a_i \in \mathbb{R}$. From [11],

$$H_L(z) = \frac{-[c_0 + c_1 z^{-1} + \ldots + c_{r-1} z^{-(r-1)}]}{1 + a_1 z^{-1} + \ldots + a_{q-1} z^{-(q-1)} + a_q z^{-q}} \cdot z^{-L} \tag{4}$$

where

$$\begin{aligned}
c_0 &= -\left[a_1 h(L-1) + a_2 h(L-2) + \ldots + a_r h(L-r))\right] \\
c_1 &= -\left[a_2 h(L-1) + a_3 h(L-2) + \ldots + a_r h(L-r+1)\right] \\
c_2 &= -\left[a_3 h(L-1) + a_4 h(L-2) + \ldots + a_r h(L-r+2)\right] \\
&\vdots \\
c_{r-2} &= -\left[a_{r-1} h(L-1) + a_r h(L-2)\right] \\
c_{r-1} &= -\left[a_r h(L-1)\right].
\end{aligned}$$

Definition 3: Let the *truncated filter* (FIR filter) be

$$\begin{aligned}
h_T(z) &= H(z) - H_L(z) \\
&= z^{-L} \cdot \left(H(z) \cdot z^L\right)_-
\end{aligned} \tag{5}$$

where $(\cdot)_-$ is only the *noncausal* portion of the impulse response.

Definition 4: Let $f^-(z)$ be the *"time-reversal filter"*

$$f^-(z) = h_T^\star(z) \cdot z^{-4L}. \tag{6}$$

From Fig. 2, it follows that

$$\frac{Y(z)}{R(z)} = h_T^\star(z) \cdot h_T(z) z^{-4L} \tag{7}$$

$$= |h_T(z)|^2 z^{-4L}. \tag{8}$$

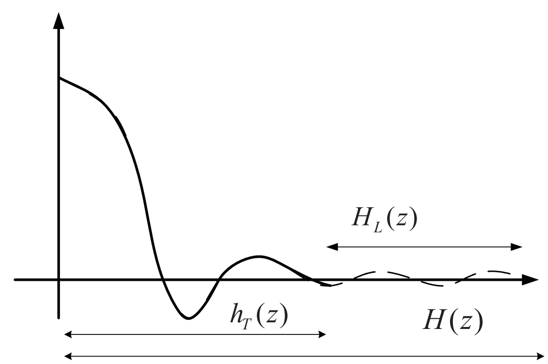

Fig. 1. Truncation of infinite impulse response at L^{th} sample.

Although this implementation is actually an FIR filter, it is termed as a "perfectly linear phase IIR filter" since it uses IIR realizations and also to remain consistent with [11]. Notice that as $L \to \infty$, then $h_T(z) \to H(z)$ and (8) $\to |H(z)|^2 \cdot z^{-4L}$. The length of L will change the magnitude characteristics of the frequency response. $H_L(z) \cdot z^L$ is typically on the order of $H(z)$ (i.e. an increase in multipliers). Throughout this paper $f^-(z)$ will also be referred to as the "Time-Reversal Filter", and when appropriate it will be mentioned as the "Inversion Filter" in Section II-C.

C. Inversion of Non-Minimum Phase Zeros

For stable minimum phase systems, direct pole-zero cancelation is easiest. However for stable non-minimum phase systems, the direct cancelation of NMPZs would result in an unstable controller. The approximate inverse of NMPZs can be performed through a high-order FIR filter through deconvolution or equalization. Long FIR filters are costly in terms of number of multipliers and additions, especially at high sampling frequencies, when compared to IIR filters [12]. Reusing FIR filters would significantly reduce the maximum servo rate. The proposed inversion is comparable in delay to a FIR filter inversion filter, if not more. An advantage of using IIR filters is the reduced number of multipliers/additions enabling faster sampling/servo frequencies.

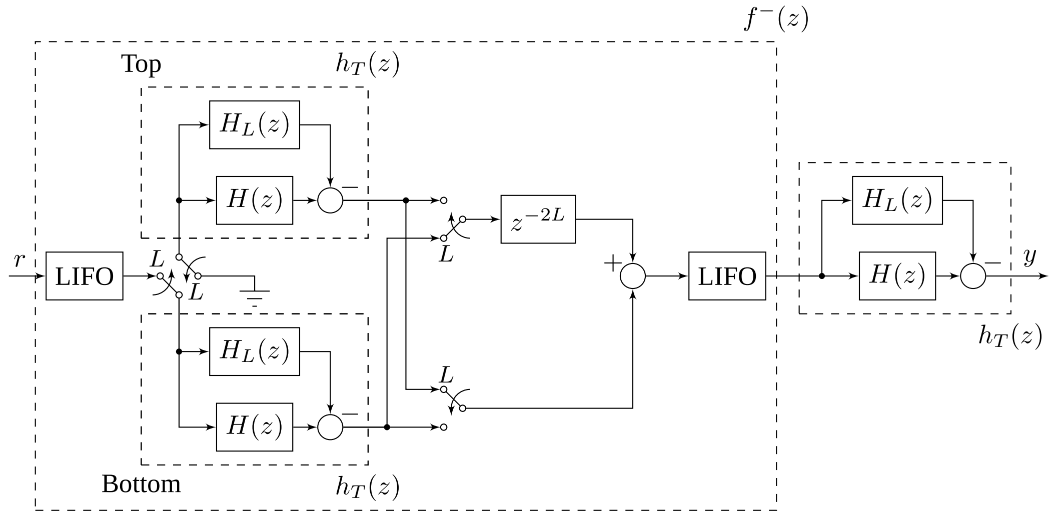

Fig. 2. Kurosu's exact linear phase filter.

Given some stable (or closed-loop stabilized) linear time-invariant (LTI) system,

$$G(z) = z^{-d} \cdot \frac{b^+(z)b^-(z)}{a(z)} \tag{9}$$

where $a(z)$, $b^+(z)$, and $b^-(z)$ are the stable poles, stable zeroes, and unstable zeros, respectively. Let the proposed inversion filter be $F(z) = f^-(z) \cdot F^+(z)$ as illustrated in Fig. 3.

Definition 5: Define $deg(\cdot)$ as the degree or order of the polynomial (i.e. number of roots).

Definition 6: Define the constants ρ

$$\rho = deg(b^-(z)). \tag{10}$$

For this case, let $F^+(z)$ be the inversion of stable poles/zeros where

$$F^+(z) = \frac{a(z)}{b^+(z)} M \tag{11}$$

By letting $H(z)$ inside $f^-(z)$ be

$$H(z) = \frac{1}{b^{-\star}(z)} M \tag{12}$$

approximate inversions of non-minimum phase zeros is possible. Since most physical systems are bandlimited, $M(z)$ can be chosen to be the reference model, which limit the bandwidth of the inversion to avoid large gains at the high frequency regions. Similarly, $M^\star(z)$ needed such that the reference model is linear phase. For simplicity, we will assume $M(z) = M^\star(z) = 1$. Ideally, a stable inversion is desired such that $|f^-(z)| \approx \left|\frac{1}{b^-(z)}\right|$. Using Fig. 2 and (5) and the above choice of $H(z)$ results in

$$h_T(z) = \frac{1}{b^{-\star}(z)} - \left[\frac{1}{b^{-\star}(z)}\right]_L . \tag{13}$$

Notice if L is long enough, then

$$f^-(z) = h_T^\star(z) z^{-4L} \approx H^\star(z) z^{-4L} \tag{14}$$

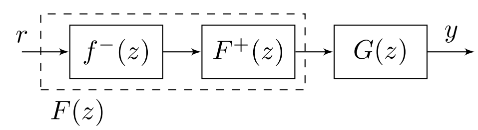

Fig. 3. Realization of approximate NMPZ inversion.

Let $H(z) = \frac{1}{b^{-\star}(z)}$, then

$$H^\star(z) z^{-4L} = \left(\frac{1}{b^{-\star}(z)}\right)^\star z^{-4L} \tag{15}$$

$$= \frac{1}{b^-(z)} z^{-4L}. \tag{16}$$

Thus

$$f^-(z) = \left(\frac{1}{b^-(z)}\right)_T z^{-4L} \approx \frac{1}{b^-(z)} z^{-4L}. \tag{17}$$

Recall $f^-(z)$ uses the Kurosu filter (Fig. 2) to realize an approximate inverse since in a standard filter form it is unstable. Letting $H(z)$ be the inverse of the mirrored NMPZs will produce an approximate inversion. This means that as $L \to \infty$, $f^-(z) \cdot F^+(z) \to H^{-1}(z) \cdot z^{-4L}$. In summary, Kurosu's filter allows us to implement a L^{th}-order FIR inversion filter for the NMPZs using only a few IIR filters. This results in $F(z)G(z) \approx z^{-(4L+d)}$.

III. Delta Operator

The Delta Operator is defined as

$$\delta^{-1} = \frac{\Delta q^{-1}}{1 - q^{-1}} \tag{18}$$

where q^{-1} is the shift operator in the time-domain. The equivalent frequency domain representation is

$$\gamma^{-1} = \frac{\Delta z^{-1}}{1 - z^{-1}}, \tag{19}$$

where Δ is the sampling period [13]. The block-diagram filter realization for the Delta Operator is shown in Fig. 4.

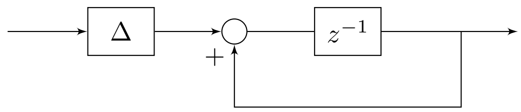

Fig. 4. Implementation of Delta Operator - γ^{-1}

Fig. 5 shows that a controller can be broken into series of second order transfer functions/filters defined as

$$H_k(z) = \frac{b_{0,k} + b_{1,k}z^{-1} + b_{2,k}z^{-2}}{1 + a_{1,k}z^{-1} + a_{2,k}z^{-2}} \tag{20}$$

where k denotes the k^{th} second-order section (SOS).

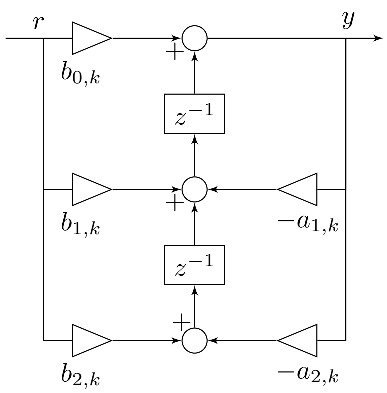

Fig. 5. Direct Form II Transposed Implementation

A mapping of $H_k(z)$ into $H_k(\gamma)$, exists as

$$H_k(z)\Big|_{z=1+\Delta\gamma} = H_k(\gamma). \tag{21}$$

The resulting Delta Operator SOS would be

$$H_k(\gamma) = \frac{\beta_{0,k} + \beta_{1,k}\gamma^{-1} + \beta_{2,k}\gamma^{-2}}{1 + \alpha_{1,k}\gamma^{-1} + \alpha_{2,k}\gamma^{-2}} \tag{22}$$

Figure 6 shows how the DFIIt structure is similar where the shift operator, z^{-1}, is replaced by the Delta Operator, γ^{-1}. There exists a relationship between z-domain coefficients and γ-domain coefficients shown in Table I and II. Δ does not necessarily have to be the sampling period since the Δ values in γ^{-1} and coefficients β_k, α_k cancel each other out [14]. To save on multipliers, Δ can be chosen to be a power of 2 which can be efficiently realized as simple bit shifts.

β_0	b_0	α_0	1
β_1	$\frac{b_0+b_1}{\Delta}$	α_1	$\frac{1+a_1}{\Delta}$

TABLE I

DELTA OPERATOR COEFFICIENT MAPPING FOR FIRST ORDER FILTER

β_0	b_0	α_0	1
β_1	$\frac{2b_0+b_1}{\Delta}$	α_1	$\frac{2+a_1}{\Delta}$
β_2	$\frac{b_0+b_1+b_2}{\Delta^2}$	α_2	$\frac{1+a_1+a_2}{\Delta^2}$

TABLE II

DELTA OPERATOR COEFFICIENT MAPPING FOR SECOND ORDER FILTER

The Delta Operator can be viewed as the forward difference mapping of the unit circle, in the z-domain, and

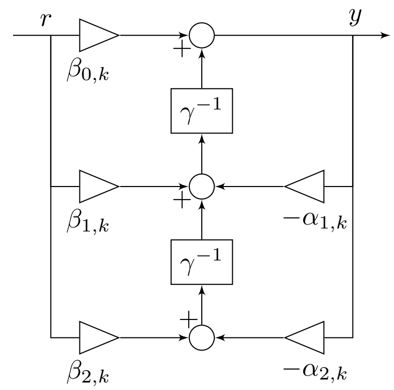

Fig. 6. Direct Form II Tranposed Implementation - Delta Operator

mapping it into a pseudo s-domain (continuous time) domain. Results from [13] show that with sufficiently high-sampling rate, 12-bit Delta Operator representation is capable of significantly lower quantization noise than a 12-bit shift operator realization. In additiion, for every delay in the DFIIt form, the Delta Operator form uses one extra addition which is inexpensive compared to a multiplier. The Delta Operator is attractive due to its potentially substantial decrease of quantization noise comes at a slight increase in computational complexity [16]. The use of the Delta Operator seems to be a very useful in the cases of high-sampling rate control on a FPGA as in Section IV.

IV. REPETITIVE CONTROL OF A PIEZOELECTRIC DEVICE

A. System Identification

The experimental example used for this paper is a piezoelectric cutting tool actuator designed for dynamic variable depth of cut machining [17]. In addition to being mechanically preloaded to reduce hysteresis, a discrete-time PI controller, $C(z)$, was added to prevent position drift of the open loop system, $P(z)$, during regulation and to reduce the magnitude of the resonant peak of the open-loop system.

The identified closed-loop system in Fig. 7, $G(z) = CP/(1+CP)$, was excited with a pseudo random binary sequence (PRBS) at $100kHz$ sampling rate. The following input-output relationship was obtained through Prediction Error Method (PEM) type system identification techniques

$$G(z) = \frac{-0.00020352(z-7.577)(z-0.8446)}{(z-0.9707)(z^2-1.849z+0.8812)} \times \frac{(z^2-1.724z+0.9237)(z^2-3.222z+3.559)}{(z^2-1.885z+0.9829)(z^2-1.214z+0.8176)}. \tag{23}$$

B. Repetitive Control

Figure 8 illustrates a simple RC plug-in structure where $F(z)$ and $F(\gamma)$ is a type of feedforward inversion of the closed-loop plant $G(z)$ [4], [18]. Other variations of RC include the ZPETC based repetitive control structure [4], [19]. $F(z)$ is constructed through the methods of Section II.

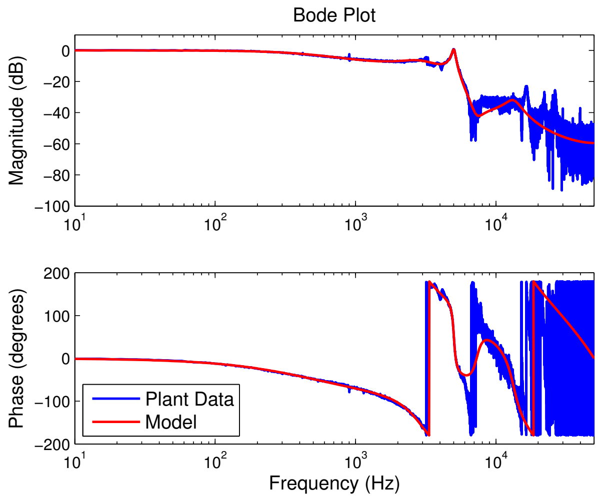

Fig. 7. Plant Data vs. Model

The value of L was chosen to be 50. For $F(\gamma)$, techniques from [14] were used to choose appropriate power-of-2 values for Δ.

N_1 and N_2 are chosen such that $N_1 + N_2 + N_q = N$ and $N = f/f_s$. f_s is the sampling frequency, f is the fundamental frequency of the periodic reference or disturbance, and N_q is the equivalent linear phase delay introduced by the low pass filter $q(z)$. RC will track and reject the fundamental frequency and all of its harmonics of the reference and disturbance, respectively. For our application, the sampling frequency was chosen such that $f_s = 100kHz$. The high sampling rate is necessary to track a $1kHz$ triangular wave. A base frequency of $250Hz$, $N = 400$, to accommodate the long delay from $F(z)$ or $F(\gamma)$ and since 250 is common factor of 1000.

$q(z)$ was chosen to be $0.25 + 0.5z^{-1} + 0.25z^{-2}$ with an equivalent phase delay of z^{-N_q} with $N_q = 1$. $q(z)$ is a linear phase low pass filter where the coefficients are represented as efficient bit shifts instead of multipliers. The $q(z)$ in the repetitive controller serves to help stabilize the system and can be seen as a frequency dependent learning gain [2]. Given the choice of $q(z)$ and L, then $N_2 = 4L + \rho + d = 200+3+1 = 204$ and $N_1 = N - N_2 - N_q = 400-204-1 = 195$.

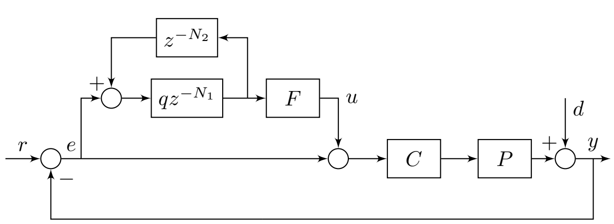

Fig. 8. RC with feed-forward inversion and PI controller.

C. Experimental Results

A Virtex 5 based National Instruments FPGA PCIe-7852 was used to implement both, the DFIIt and Delta Operator, forms of a repetitive controller. All filter (controller) coefficients were implemented using a 16-bit representation while signals were represented with 32-bit to ameliorate overflow/saturation effects. A sampling rate of $100kHz$ was used to track high frequency periodic references for both the DFIIt and Delta Operator implementation.

Figure 9 and 10 compares the experimental results when tracking a $1kHz$ triangular reference. The reference tracking plot in Fig. 9 shows adequate performance for both the DFIIt and Delta Operator realization. The error signals in Fig. 9 show that both filter forms approach their respective quantization noise floor. As seen in [8], quantization noise floor prevents the error from reaching absolute zero. As predicted, the Delta Operator error is much lower than the DFIIt realization's error. The control signal, u, of Fig. 9 refers to only the contribution of the repetitive controller portion (output of $F(z)$). We can see in the "control signal" that the signal is not purely periodic but laced with quantization noise. This order of error reduction is crucial in clean atomic force microscope applications [1]. Figure 10 shows the power spectral density (PSD) of the error signal of the DFIIt and Delta operator realization. A majority reduction of the quantization noise by the Delta Operator, compared to DFIt, can be seen in in low frequency bands since the q-filter suppresses noise at high frequencies.

Quantization noise is typically a function of both the filter (controller) and the reference signal. Table III lists the max error, $|e_{max}|$, and RMS error, e_{RMS}, for reference signals with varying magnitudes and frequencies. In the Delta Operator realization, the errors grow linearly with respect to the magnitude of the reference. However, this is not the case in the DFIIt realization where the errors have become much larger than expected from the linear growth as the reference magnitude increases beyond some level. At small magnitudes, such as regulation of $0\mu m$ reference, the e_{RMS} of Delta and DFIIt are comparable but $|e_{max}|$ of DFIIt is significantly larger. Overall, the general trend shows that the errors $|e_{max}|$ and e_{RMS} is almost always smaller in the Delta Operator form than the DFIIt for this experiment.

TABLE III

REPETITIVE CONTROL PERFORMANCE WITH DELTA OPERATOR AND DFIIT.

		e_{RMS} (μm)		$\lvert e_{max}\rvert$ (μm)	
	Triangular Reference	Delta	DFIIt	Delta	DFIIt
	$0\mu m$	0.0247	0.0251	0.1158	1.0675
$1kHz$	$\pm0.50\mu m$	0.0288	0.0799	0.1460	2.0847
	$\pm1.01\mu m$	0.0367	0.0989	0.2014	2.2609
	$\pm2.01\mu m$	0.0589	0.3259	0.3172	3.2025
	$\pm4.03\mu m$	0.1087	2.4832	0.5338	7.6437
	$\pm8.06\mu m$	0.2190	4.5914	0.9869	11.0728
$2kHz$	$\pm0.52\mu m$	0.0596	0.0869	0.1964	2.1350
	$\pm1.04\mu m$	0.1105	0.2235	0.2870	3.1068
	$\pm2.09\mu m$	0.2164	1.7736	0.4985	6.4806
	$\pm4.19\mu m$	0.4260	3.0222	0.8812	8.7616

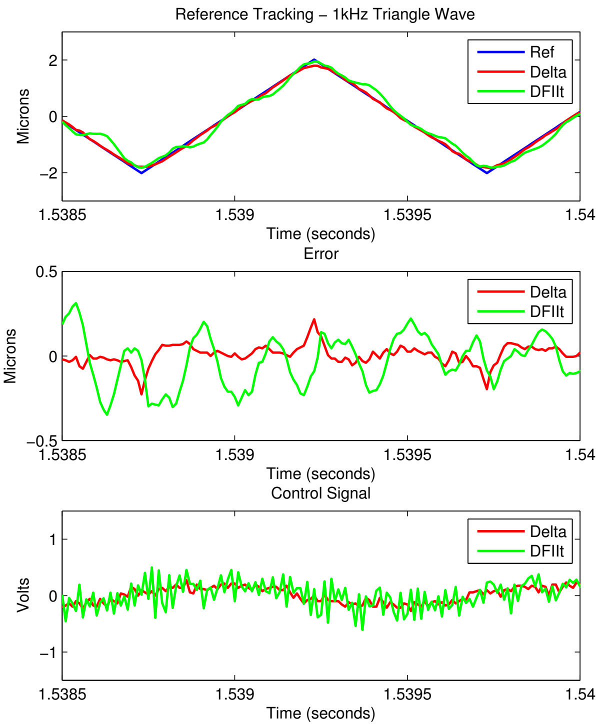

Fig. 9. Reference Tracking and Error of 1kHz Triangular Wave.

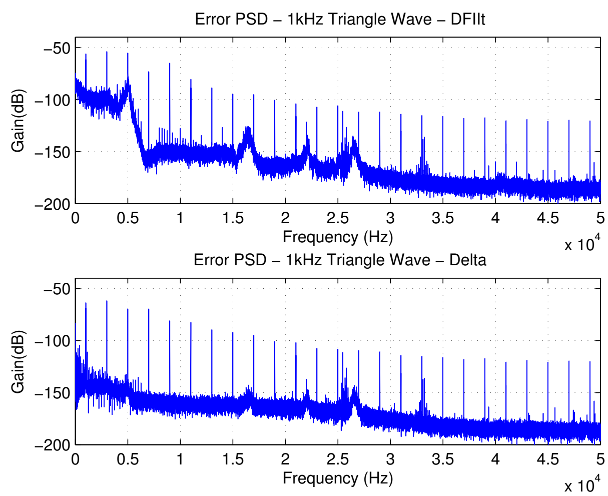

Fig. 10. Error PSD with 1kHz Triangular Wave Reference Tracking.

V. Conclusion

We have realized our previously formulated efficient NMPZ inversion and RC structure by the Delta Operator on a FPGA and established a still efficient repetitive control structure. The experimental results for controlling a piezoelectric actuator show that the improvement of Delta Operator over the DFIIt is significant. Since our FPGA realization does not reuse the computation resources to serialize the digital signal processing during the controller update, the sampling rate of the controller can be increased to near the FPGA clock speed, $40MHZ$ in our case, if it is called for in other applications. At such high rate, it would be unlikely to realize high-order controllers, such as the inversion based repetitive control, without exploiting the efficient digital signal processing techniques discussed in this paper.

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