Efficient Fixed-Point Realization of Approximate Dynamic Inversion Compensators for Non-Minimum Phase Systems

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***Abstract*— This paper considers a fixed-point realization of approximate stable inversion of systems with non-minimum phase zeros. It is based on the efficient linear phase IIR filter implementation first introduced by Powell and Chau. The proposed filter is used in inversion based feedforward tracking and repetitive controllers which is realized by a Field Programmable Gate Array to control a piezoelectric actuator and generate precise dynamic high bandwidth motion at a 100 kHZ sampling rate.**

I. INTRODUCTION

The presence of high bandwidth systems such as scanning probe microscopy and micro-electro-mechanical (MEMS) actuators warrants the need for high sampling rate control systems.

The goal of this paper is to generate precise scan trajectories at the kHz regime by feedforward tracking and repetitive control with a sampling rate of 100 kHz. The realization of the controllers hinge upon implementing a proposed real-time approximate stable inversion of non-minimum phase systems on Field Programmable Gate Arrays (FPGAs), which uniquely suit the application at hand due to parallel computing, speed, and low level interface to physical systems.

Our proposed stable inversion compensator stems from previous work on linear phase IIR filters created for efficient real-time implementation. Powell & Chau first introduced the concept of a real-time implementation of a linear phase IIR filter [1]. The realization involves L-length localized time reversals, overlap-add sectioned-convolutions, and another set of time reversals. The end-result produces a linear phase IIR filter with phase equal to that of z^{-4L}. The reset used to truncate the impulse response resulted in parasitic sinusoidal phase disturbances. Thus, resulting in only an approximately linear phase IIR filter. Later, Kurosu [2] proves the imperfections of the Powell-Chau filter analytically and modifies Powell & Chau's structure. Kurosu's modified Powell-Chau filter can be proven to have no phase disturbances. Kurosu exploits the fact that any FIR filter can be represented as the subtraction of two IIR filters. By combining that with the Powell-Chau filter, Kurosu introduces a perfectly linear phase IIR filter. An FIR filter realizing the identical input-output relation would require many more multipliers and adders, up to one or two orders of magnitude, than that of the modified Powell-Chau filter.

In repetitive control, zero phase error compensation [3], which performs stable pole zero cancelation and conjugate compensation on non-minimum phase zeros has been used for approximate inversion in the controller design. This simple compensator is a form of linear phase stable inversion but has a limitation in lack of control over dynamic range or the inversion over the desired frequency range. In Iterative Learning Control (ILC) literature, Ghosh and Paden [4] presented similar time reversal techniques for continuous time offline inversion of nonlinear non-minimum phase plants. However, this off-line technique for ILC is not applicable to real-time repetitive control algorithms. The contribution of this paper is the formulation and implementation of a real-time linear phase inversion of non-minimum phase systems through the manipulation of Kurosu's filter implementation.

The reminder of this paper is structured as follows: Section II provides some background and motivation for the proposed structure. We analyze the complications of high-sampling rate with fixed-point computation when non-minimum phase zeros are present. Furthermore, dynamic range of filter coefficients will play a role of fixed-point realized controllers. Section III describes the implementation of Kurosu's filter and a basic understanding of linear phase IIR filters. Section IV formulates approximate inversions of non-minimum phase zeros. Section V places the proposed inversion filter within the repetitive control framework. Experimental results on a piezoelectric actuator are presented.

II. BACKGROUND AND MOTIVATION

The implementation of high sampling rate controllers on a fixed-point platform may cause some issues. It is useful to have an understanding of some of these issues before looking at the proposed filter.

A. Large Dynamic Range

An issue accompanying high-sampling rates is the need for high precision as poles/zeros crowd towards the unit circle. In terms of fixed-point, dynamic range is sacrificed for high precision.

Let

$$H(z) = \frac{b_0 + b_1 z^{-1} + \ldots + b_{M-1} z^{-(M-1)} + b_M z^{-M}}{a_0 + a_1 z^{-1} + \ldots + a_{N-1} z^{-(N-1)} + a_N z^{-N}} \quad (1)$$

where $N \geq M$.

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1) Coefficient Range: Assuming Direct Form II, a loose approximation of the minimum number of bits is as follows:

$$\rho_1 = max\left[\left(\frac{max(|a_i|)}{min(|a_j|)}, \frac{max(|b_k|)}{min(|b_l|)}, \frac{max(|b_k|)}{max(|a_i|)}\right)\right] \quad (2)$$

$$\#\text{ of bits} \geq \lceil \log_2(\rho_1) + 1 \rceil \quad (3)$$

where $i, j = 1, \ldots, N$ and $k, l = 1, \ldots, M$.

The above shows a lower bound on the minimum number of bits necessary for the smallest dynamic coefficient range and for the best precision. This is a loose lower bound because it does not account for the range of the input signal. With a finite word-length, the output of the filter would surely saturate/wrap. Thus, the actual number of bits would increase or precision would have to be sacrificed.

2) Large Filter Dynamic Range: Depending on the input signal, the filter will require more bits due to the P-norm scaling as addressed in [5]. Scaling and number of lower-order cascaded sections used to realize a fixed point filter, will affect filter performance as well. Scaling depends on which norm is chosen:

$$||H(i\omega)||_p = \left[\frac{1}{\omega_s}\int_0^{\omega_s} |H(i\omega)|^p d\omega\right]^{\frac{1}{p}} \quad (4)$$

where ω_s is the sampling frequency in rad/s. For this paper we choose $p = \infty$ to prevent numerical overflow. ρ_2 is the number of bits necessary to compensate for the input and output signals.

3) Precision Bits: After using the necessary bits to ensure coefficient range and adjusting for P-norm scaling, the rest of the bits left over are used for the precision of the coefficients.

$$\text{Precision Bits} = Y - \rho_1 - \rho_2 - 1 \quad (5)$$

where Y is the word-length used in hardware(e.g. 16 bits). Equation 5 is only a rough estimation of the number of precision bits assuming coefficients and input/output signals representations are equal.

III. Linear Phase IIR Filter - Modified Powell-Chau Filter

To fully appreciate the realization of the real-time linear phase IIR filter, we must first understand both the real-time FIR case, and the off-line IIR case. From there, the modified Powell-Chau Filter will become clear.

A. Linear Phase Filters

The most common linear phase filters are FIR filters typically with symmetric taps (coefficients) [6]. In order to make any FIR filter $D(z)$ linear in phase, we can cascade it with $D^\star(z)$(i.e. $T(z) = D(z) \times D^\star(z)$). The resulting $T(z)$ will have linear phase with magnitude response of $|D(z)|^2$ (which was done in Tomizuka's paper [3]). $T(z)$ is noncausal but can be compensated by a certain number of delays. Note that $D^\star(z)$'s zeros are mirrored images of $D(z)$'s zeros. "Mirrored" in this context is with respect to the unit circle. For IIR filters to be linear phase both their zeros and poles must have mirrored pairs. Thus, IIR filters are not possible as their conjugates are unstable.

B. Truncation of IIR Filter

Any FIR filter can be represented as the difference between 2 IIR filters. $H_L(z)$ is the remaining impulse response after $H(z)$ has been truncated at time sample L as seen in Figure 1. Given that $H(z)$ is

$$H(z) = \frac{b_0 + b_1 z^{-1} + \ldots + b_{M-1} z^{-(M-1)} + b_M z^{-M}}{a_0 + a_1 z^{-1} + \ldots + a_{M-1} z^{-(M-1)} + a_M z^{-M}} \quad (6)$$

and $h(n)$ represents the impulse response at time step n. The exact relation for $H_L(z)$ has been presented by Kurosu [2] ,which is:

$$H_L(z) = \frac{-[c_0 + c_1 z^{-1} + \ldots + c_{M-1} z^{-(M-1)}]}{a_0 + a_1 z^{-1} + \ldots + a_{M-1} z^{-(M-1)} + a_M z^{-M}} \quad (7)$$

where,

$$c_0 = b_1 h(L-1) + b_2 h(L-2) + \ldots + b_M h(L-K))$$
$$c_1 = b_2 h(L-1) + b_3 h(L-2) + \ldots + b_M h(L-K+1)$$
$$c_1 = b_3 h(L-1) + b_4 h(L-2) + \ldots + b_M h(L-K+2)$$
$$\vdots$$
$$c_{M-2} = -[b_{M-1} h(L-1) + b_M h(L-2)]$$
$$c_{M-1} = -[b_M h(L-1)]$$

The truncated filter is $H_T(z) = H(z) - H_L(z)z^{-L}$. To save computational resources, it is possible to perform a model reduction on $H_L(z)$. Also, note that as L increases, $H_L(z)$ tends to become near quantization level and thus is not necessary.

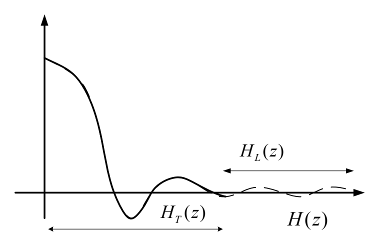

Fig. 1: Truncation of IIR Impulse Response at length L

C. Realization of Modified Powell-Chau Filter

The realization of Kurosu's modified filter [2] is shown in Figure 2. L is chosen such that the remaining portion of the length of the impulse response is near quantization level. Also, we assume that $H(z) = H_{top}(z) = H_{bot}(z)$ for simplicity. For clarification, we will refer to the conjugate filter implementation as the modified Powell-Chau filter or P(z).

Last-In-First-Out (LIFO) memory units are utilized to perform the localized L-word time reversal. In order it for to

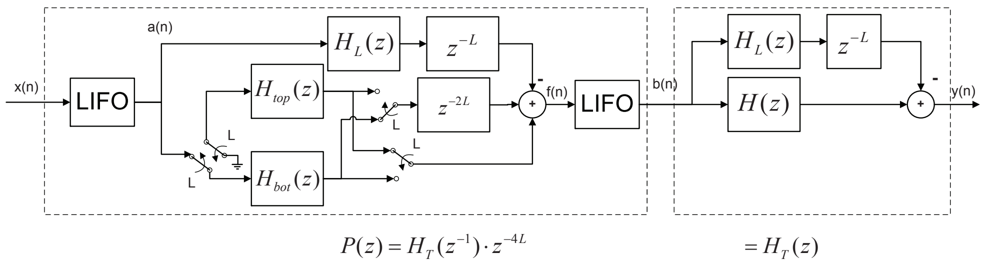

Fig. 2: Realization of Kurosu's Exact Linear Phase IIR Filter

work, each LIFO must have a z^{-L} delay to remain causal. Detailed implementation and operation of the L-word LIFO can be found in [1].

Kurosu's has the same fundamental idea as the offline linear phase IIR filter with respect to time reversal. Switching is used to help perform the overlap-add convolution and makes the process real-time. Furthermore, if the remaining impulse response is at quantization level then $H_L(z)$ does not need to be realized.

D. Input-Output Relation of Kurosu's Linear Phase IIR filter

Let us first define some notation. Observe the zero-pole-gain form,

$$H(z) = \bar{k}\frac{(z+z_1)(z+z_2)\dots(z+z_r)}{(z+p_1)(z+p_2)\dots(z+p_q)} \tag{8}$$

where, $q \geq r$. Then its H-conjugate filter is defined as,

$$H^{\star}(z) = \underbrace{\left(\bar{k}\frac{z_1 z_2 \dots z_r}{p_1 p_2 \dots p_q}\right)}_{\gamma} \frac{(z+\frac{1}{z_1})(z+\frac{1}{z_2})\dots(z+\frac{1}{z_r})}{(z+\frac{1}{p_1})(z+\frac{1}{p_2})\dots(z+\frac{1}{p_q})} \tag{9}$$

γ is used to characterize the gain of the conjugate filter. Note that when relative order of $H(z)$ is not 0, that $H^{\star}$ and $H(z^{-1})$ are not the same. $H(z^{-1})$ is $H(z)$ with z's replaced by z^{-1}. The two differ by a gain γ.

Since [2] has proven linearity and time invariance, it follows that there is an exact relation to the modified Powell-Chau filter. Exact input-output relation can be retrieved from Figure 2 detailed by

$$H_T(z) = H(z) - H_L(z)z^{-L} \tag{10}$$

$$\frac{Y(z)}{X(z)} = H_T(z^{-1})H_T(z)z^{-4L} \tag{11}$$

$$= ||H_T(j\omega)||^2 z^{-4L} \tag{12}$$

where, $H_L(z)$ is defined as the truncated portion of the impulse response of $H(z)$. $H_T(z)$, $H_T(z^{-1})$ and $H_T(z) \cdot H_T(z^{-1})$ are FIR filters but implemented by IIR filters. This fact will become very useful when it comes to proposing our inversion filter.

If $L \rightarrow \infty$, then $H_T(z) \rightarrow H(z)$, $H_T(z^{-1})z^{-4L} \rightarrow H(z^{-1})z^{-4L}$ and $H_L(z) \rightarrow 0$. However, if $H^{\star}(z)$ implemented in filter form will be unstable. This implies that Kurosu's filter has the ability to approximate an unstable filter. We will use this to our advantage in inverting non-minimum phase zeros.

IV. Proposed Inversion Filter

A simple approximate inverse of non-minimum phase zeros is through approximating the inverse with a long FIR filter [7]. As seen by [6], long FIR filters are costly in terms of number of multipliers and additions when compared to IIR filters. Unfortunately, IIR inversions of non-minimum phase zeros are unstable. The proposed implementation is compromise between an FIR and IIR filter. The proposed inversion is comparable in delay to the FIR inversion filter, if not more. However, the main advantage is that the number of multipliers/adders could be reduced by an order of magnitude. From an implementation stand point in both floating-point and fixed-point, the reduction of multiplications/additions enables shorter sampling periods or higher sampling rates. *The conditions for the inversion to work properly is that the plant is stable and that zeros are not at* $z = 1$. The feedforward structure can be seen in Figure 3. Different choices of P and C are detailed below and in Table I.

A. Choice of $H_{top}(z)$ and $H_{bot}(z)$

For any given plant,

$$G(z) = k\frac{B^{+}(z)B^{-}(z)}{A(z)} \tag{13}$$

where $A(z)$, $B^{+}(z)$, and $B^{-}(z)$ are the stable poles, stable zeroes, and unstable zeros, respectively. Let the proposed inversion controller be $F(z) = P(z)C(z)$. Depending on the choice of $C(z)$, $Y(z)/R(z)$ will be different as seen in Table I.

1) Proposed Inversion: $C(z)$ is the inversion of stable poles/zeros, assuming a stable plant $G(z)$.

$$C(z) = \frac{A(z)}{kB^{+}(z)z^{-D}} \tag{14}$$

$$D = deg(A) - deg(B) \tag{15}$$

$$D_H = \text{relative order}(H_{top}) \tag{16}$$

where $deg(\cdot)$ stands for order (or number of roots). By letting $H_{top}(z)$ and $H_{bot}(z)$ inside $P(z)$ be equal to $\frac{\gamma}{B^{-\star}(z)}$, approximate inversions of non-minimum phase zeros is possible. Ideally, a stable inversion is desired

$$P(z) = H_T(z^{-1})z^{-4L} \tag{17}$$

$$\approx \frac{1}{\bar{k}} \cdot \frac{1}{B^{-}(z)}z^{-4L} \tag{18}$$

Controller	H(z) =	C(z) =	Y(z)/R(z)
Approx. Inv.	$\frac{\gamma M_2}{B^{-\star}(z)}$	$\frac{A(z)M_1}{kB^{+}(z)z^{-D}}$	$\approx z^{-4L-D+D_H}$
IIR-ZPETC	$G(z)$	1	$\approx \|\|G(z)\|\|^2 z^{-4L}$
ZPETC	$\frac{B^{-}(z)}{kB^{-}(1)^2}$	[illegible]	$= \frac{\|\|B^{-}(z)\|\|^2}{B^{-}(1)^2} \cdot z^{-4L-D+D_H}$

TABLE I: Choices of $H(z)$ inside $P(z)$ filter and $C(z)$

Recall from the previous section, $P(z)$ can not be realized in any standard filter form since it is unstable. However, using the modified Powell-Chau Filter as seen in Figure 3, we can obtain an approximate inverse. However, letting $H(z)$ be the inverse of the mirrored unstable zero as illustrated in Figure 3 will produce an approximate inversion. Then, $C(z)$ will invert all stable poles/zeros.

For some certain applications, it would be better to have the system to be band-limited. It is easy to add a reference model to the above proposed inversion. More generally, $C(z) = \frac{A(z)}{kB^{+}(z)z^{-D}}M_1$, and $H_{top}(z) = H_{bot(z)} = \frac{\gamma M_2}{B^{-\star}}(z)$ where M_1 can be your reference model and M_2 can be chosen to give linear phase characteristics. Certainly, one may retain certain poles/zeros rather than canceling them to save on computation. For simplicity, the values $M_1 = M_2 = 1$ are used in the rest of the paper. Using Equation 10 and 12,

$$H_T(z) = \frac{\gamma}{B^{-\star}(z)} - \left[\frac{\gamma}{B^{-\star}(z)}\right]_L \cdot z^{-L} \tag{19}$$

2) Special Case: IIR-ZPETC: Another possible choice is $H_{top}(z) = H_{top}(z) = G(z)$ and $C(z) = 1$. Since this $G(z)$ is not an FIR filter, $P(z) \approx G(z^{-1})z^{-4L}$. This is simply Kurosu's filter substituted with a physical plant, $G(z)$. We will denote this choice of $H(z)$ and $C(z)$ as the IIR-ZPETC as $Y(z)/R(z)$ is linear-phase (i.e. $Y(z)/R(z) = ||G(i\omega)||^2 z^{-4L}$). However, the major drawback of this method is that if $G(z)$ were to have low-pass characteristic, as most physical system do, the bandwidth would probably be much more limited than the traditional ZPETC. In other words, the magnitude characteristic of $||G(j\omega)||^2$ may not be desirable.

3) Special Case: ZPETC: It is interesting to note that if $H_{top}(z) = H_{bot}(z) = \frac{B^{-}(z)}{kB^{-}(1)^2}$ and $C(z) = \frac{A(z)}{B^{+}(z)}$, we will obtain a delayed version of ZPETC [3]. Since $B^{-}(z)$ is a FIR filter, then if L chosen to be the order of $B^{-}(z)$ then $P(z) = B^{-}(z^{-1})z^{-4L}$. This is simply an academic observation, and the modified Powell-Chau probably would not be used in this case since it is more practical to implement the standard ZPETC. However, it does show that where ever ZPETC is used, the proposed inversion with reference model may be a viable alternative.

B. Reduction of L

The main disadvantage of the modified Powell-Chau implementation, is the $4L$ delay introduced where L is the length of the impulse response of $H(z)$ in Figure 2. However, it is important to note that P(z) *can "invert" nonminimum phase zeros and stable poles*. By inverting stable poles that are close to the non-minimum phase zeros, one could reduce the impulse response length to induce a near "pole-zero" cancelation effect. Thereby, reducing the length of L. In addition, Miyase [8] has implemented methods of reducing L at the cost of complicating P(z)'s structure (e.g. more additions/multiplications).

C. Fixed Point Dynamic Range Flexibility

Depending on the plant, the dynamic range and coefficient range of C(z) could be large. The same problem may occur to P(z), if G(z) exhibits many unstable zeros. This effect could possibly be reduced by moving the cancelation of poles from C(z) into P(z) at the slight price of increased computation.

V. Application to Repetitive Control

A. Repetitive Control

Using the plug-in repetitive control structure [9], [10], as illustrated in Figure 4, the proposed inversion filter can be used in two different places. F_1 can be used as a feedforward inversion filter. Also, F_2 can also be the proposed inversion instead of a ZPETC. In this case, $F_2 \approx \left(\frac{CG}{1+\ldots}\right)^{-1}$ [illegible]. $N_3 = 4L_1 + D - D_H$ is the delay introduced by the inversion filter F_1. $N_2 = 4L_2 + D - D_H$ is the delay introduced by the inversion filter F_2. $N_1 = N - N_2 - N_q$, where $N = \frac{f_s}{f}$ and N_q is the delay introduced by the linear phase low-pass filter Q. f_s is the sampling frequency and f is the reference frequency. A convenient choice of Q, for this application, would be $(0.25 + 0.5z^{-1} + 0.25z^{-2})$ since the coefficients do not require multiplications but rather bit shifts.

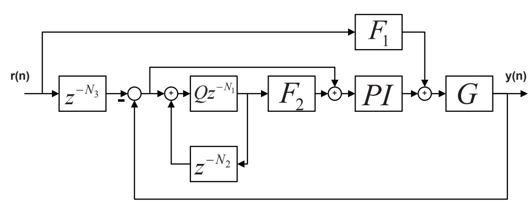

Fig. 4: Repetitive Control with FF Inversion and PI Control

B. Sensitivity Analysis

For a simple PI control:

$$S_{PI} = \frac{1}{1 + KG} \tag{20}$$

For PI control with Feedforward Inversion K:

$$S_{PI+FF} = \frac{1}{1+KG} \cdot (z^{-L} - F_1 G) \tag{21}$$

$$= S_{PI} \cdot (z^{-L} - F_1 G) \tag{22}$$

For PI control with Feedforward and Repetitive Control:

$$S_{PI+FF+RC} = \frac{(1-Qz^{-N})(z^{-L}F_1G)}{(1-Qz^{-N})(1+CG)+F_2CG(1-Qz^{N_1})} \tag{23}$$

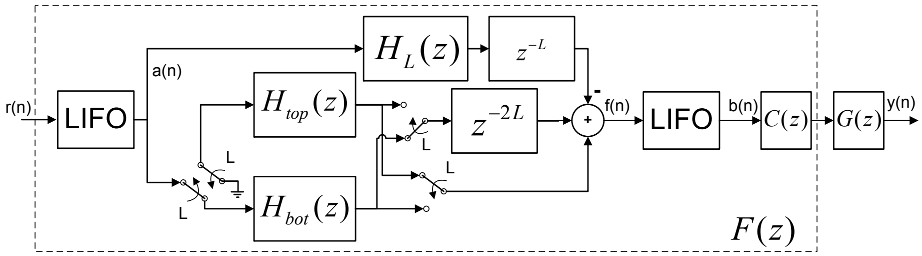

Fig. 3: Realization of Approximate Non-minimum phase-zero Inversion

Assuming F_1 and F_2 are near inversions:

$$S_{PI+FF+RC} \approx \frac{(z^{-L}-F_1G)}{(1+CG)} \cdot \frac{(1-Qz^{-N})}{(2-Qz^{-N}-Qz^{-N_1})} \quad (24)$$

$$= S_{PI+FF} \cdot \frac{(1-Qz^{-N})}{(2-Qz^{-N}-Qz^{-N_1})} \quad (25)$$

C. An Example

The plant used for the experimental study is a modified version of the piezoelectric actuated fast tool servo (FTS) [11] for noncircular lathe cutting. A system identification resulted in the following:

$$G(z) = \frac{6.36 \times 10^{-5}(z-20.15)(z^2-2.60z+1.85)(z^2-1.78z+0.97)}{(z^2-1.84z+0.86)(z^2-1.75z+0.81)(z^2-1.89z+0.99)} \quad (26)$$

where the sampling frequency is 100kHz. A lowpass filter was necessary during system identification to reduce the nonlinear effects present at higher frequencies. A simple PI controller was designed for the system:

$$K = \frac{-0.50644(z-0.9376)}{(z-1)} \quad (27)$$

An integrator is used to compensate for piezo drift along with maintaining unity gain at low frequencies. Next a feedforward controller using the proposed inversion filter was created for tracking purposes. $P_1(z)$, $P_2(z)$, $C_1(z)$ and $C_2(z)$ were chosen to reduce L. They were also chosen to adjust the dynamic range and coefficient range to fit within a 16-bit framework. For this example, L_1=L_2=20 for both F_1 and F_2. P_1,C_1 are for F_1. P_2, C_2 are for F_2.

$$P_1(z) = \frac{-4.0799(z^2-1.907z+1.011)}{(z-0.04963)(z^2-1.403z+0.5395)} \quad (28)$$

$$P_2(z) = \frac{5.7369(z^2-1.904z+1.01)}{(z-0.04963)(z^2-1.403z+0.5395)} \quad (29)$$

$$C_1(z) = 101.99\frac{(z^2-1.849z+0.8585)(z^2-1.747z+0.805)}{z^2(z^2-1.779z+0.9659)} \quad (30)$$

$$C_2(z) = 143.42\frac{(z-0.9694)(z^2-1.921z+0.9303)(z^2-1.707z+0.7666)}{z^2(z-0.9376)(z^2-1.779z+0.9659)} \quad (31)$$

The repetitive control was designed for a fundamental frequency at 1kHz and $f_s = 100kHz$. Q was chosen to be $0.25 + 0.5z^{-1} + 0.25z^{-2}$ such that N_q=1. Thus, N_1=80, N_2=19, and N_3=81.

D. Experimental Results

The controller was implemented on a Labview PXI-7833R FPGA board. An external 0.2V peak-to-peak triangular wave reference signal of 1kHz was used. The actuator displacement, measured by a capacitance probe is $16.5\mu m/V$. The experiment was conducted in three parts to compare the steady state tracking error as seen in Figure 5. First, only the PI control was used. As expected, the error still had significant "triangular" variation. Then the Feedforward inversion was turned on with the PI control. The slight harmonic error in the Feedforward can be attributed to fixed-point precision, an inaccurate plant model, nonlinearities, etc. Note that when the Repetitive Controller was "turned on" large transients are observed which are due to the accumulation of error during its "off" position. Table II confirms the superiority of the combination of the PI, Feedforward, and Repetitive Controller. Looking at the "zoomed-in" plot, the tracking error is close to "noise level". Table II compares noise levels at regulation vs. reference tracking. In this context "noise" refers to both electrical noise from the external reference and the quantization "noise" introduced by fixed point arithmetic/multiplications. The increase in quantization noise is explained by the increased number of fixed-point calculations(i.e. more complex controllers). Depending on the filter structure used, quantization noise can be reduced [6]. For this application, Direct Form II Transposed Cacascaded Second Order Sections provided the best compromise between quantization, number of bits, and number of adders/multipliers. Also, the Power Spectral Density illustrated in Figure 6 and 7 gives us a clearer understanding of how the Feedforward Inversion and Repetitive Control compares at reducing error harmonics. Figure 6 stops at 11kHz frequency since there is no further error reduction by the repetitive controller which is explained by the corner frequency of the Q filter.

E. Improving Robustness on Repetitive Control with Inversion Filter

To ensure robustness of the repetitive controller, a linear phase low-pass filter is often chosen. Symmetric filters are

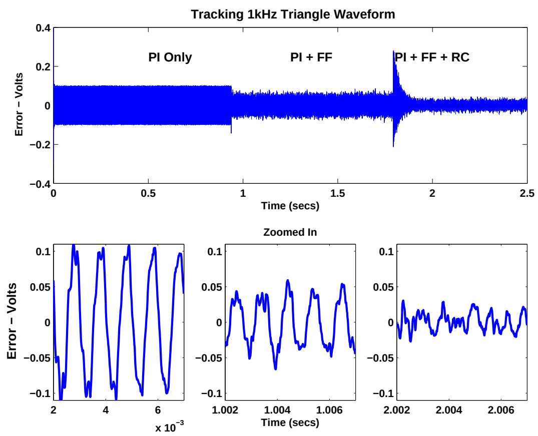

Fig. 5: Error Signal of PI, PI+FF, PI+FF+RC - Tracking 1kHz 0.2Vpp Triangle Wave

Controller	Regulation - 0V	1kHz Triangle
PI	0.0024Vrms	0.1079Vrms
PI+FF	0.0133Vrms	0.0517Vrms
PI+RC	0.0138Vrms	0.0142Vrms

TABLE II: RMS of the error for regulation and tracking

chosen for their linear phase but requires high order to obtain sharp gain drop-off. IIR filters have sharp gain-dropoff but non-linear phase. Kurosu's linear-phase IIR filter is the compromise between the two. This application is ideal for the low-pass Q filter in repetitive control.

Furthermore, we note that the inversion filter F_1 and F_2 tends to have high gains at high frequencies as well as larger uncertainties if $M_1 = M_2 = 1$. To increase the robustness of the feedback loop, M_1 and M_2 can be chosen to be a linear-phase low-pass for both F_1 and F_2. With a non-unity M_1 and M_2, F_2 can be seen as a frequency-dependent learning gain for the repetitive control.

VI. Conclusion

The modified Powell-Chau and Kurosu's filter has been exploited to construct approximate inversions of non-minimum phase systems and applied to feedforward tracking and repetitive control of a piezoelectric actuator to generate precise scanning motion. This computationally efficient approximate inversion reduces multiplications/additions while sacrificing memory size and delay steps. Although beneficial in both floating point and fixed point applications, it is best suited for high-sampling rate fixed-point realization by FPGAs as demonstrated by the experiment. The experimental results of feedforward and repetitive controllers demonstrate the effectiveness of the proposed inversion filter realization.

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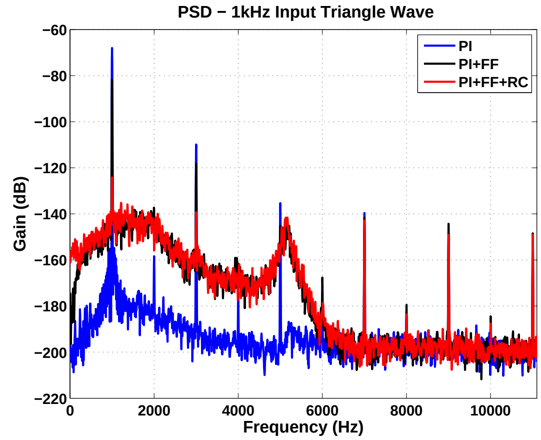

Fig. 6: Power Spectral Density of Error Signal

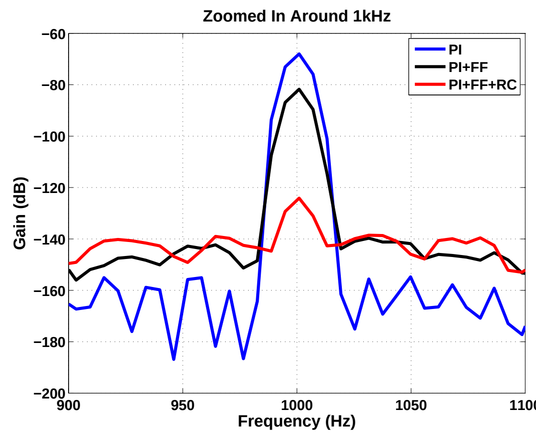

Fig. 7: Power Spectral Density of Error Signal - Zoomed In

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