Lesson - Euler's Identity

$$e^{j\theta} = \cos\theta + j\sin\theta$$

This relation is simply a transformation between rectangular coordinates and polar coordinates.

Tool Box

$$e^{j\theta} = \cos\theta + j\sin\theta$$

Example - Problem

What is X?

$$(1+j)X = (-1+j)$$

1 Solution - Method 1

Assuming X might be complex,

$$(1+j)(\alpha + j\beta) = (-1+j)$$

Expand left-side,

$$\alpha + j\alpha + j\beta - \beta = (-1+j)$$

Collect real and imaginary terms,

$$(\alpha - \beta) + j(\alpha + \beta) = (-1+j)$$

Solve system of equations

$$\alpha - \beta = -1 \quad (1)$$

$$\alpha + \beta = 1 \quad (2)$$

Adding: Eqn. 1 + Eqn. 2

$$2\alpha = 0 \quad \Longrightarrow \quad \alpha = 0$$

Subtracting: Eqn. 1 - Eqn. 2

$$-2\beta = -2 \quad \Longrightarrow \quad \beta = 1$$

Thus re-substituting α and β back in,

$$(1+j)(0+1j) = -1+j$$

$$\boxed{X = j}$$

2 Solution - Method 2

This time, use Euler's Identity to solve the same problem.

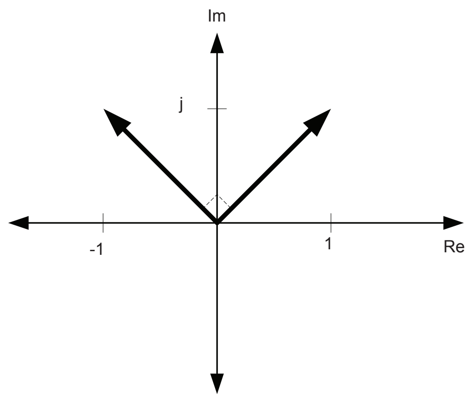

Figure 1: Figure 1

Reformulate the problem using Euler's Identity

$$(1+j)(\alpha e^{j\theta}) = (-1+j)$$

Equivalent expression using our toolbox,

$$(\sqrt{2}e^{j\frac{\pi}{4}}) \cdot (\alpha e^{j\theta}) = \sqrt{2}e^{j\frac{3\pi}{4}}$$

Solve for Magnitude first,

$$\sqrt{2}\alpha = \sqrt{2} \quad \Longrightarrow \quad \alpha = 1$$

Solve for Phase (Angle),

$$\frac{\pi}{4} + \theta = \frac{3\pi}{4} \quad \Longrightarrow \quad \theta = \frac{\pi}{2}$$

Re-substitute magnitude and phase,

$$\boxed{X = 1e^{\frac{j\pi}{2}} = j}$$